سكشن ٣ — **Fourier Transform** — اتصالات

* Transform the function from time-domain to frequency domain.

* used with non-periodic signals.

$$\begin{array}{ccc} \text{T.D} & & \text{F.D} \\ g(t) & \underset{\text{I.F.T}}{\overset{\text{F.T}}{\rightleftarrows}} & G(f) \end{array}$$

T.D: time domain

I.F.T: inverse Fourier transform

F.T

$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-J2\pi f t}\, dt$$

I.F.T

$$g(t) = \int_{-\infty}^{+\infty} G(f)\, e^{+J2\pi f t} \cdot df$$

$$\theta = \omega t$$
$$= 2\pi f t$$

Sheet #2

$\boxed{1}$ Find F.T for the rectangular Pulse shown

rectangular → rect
→ $\sqcap$

* $g(t) = A \sqcap\left(\frac{t}{\tau}\right)$

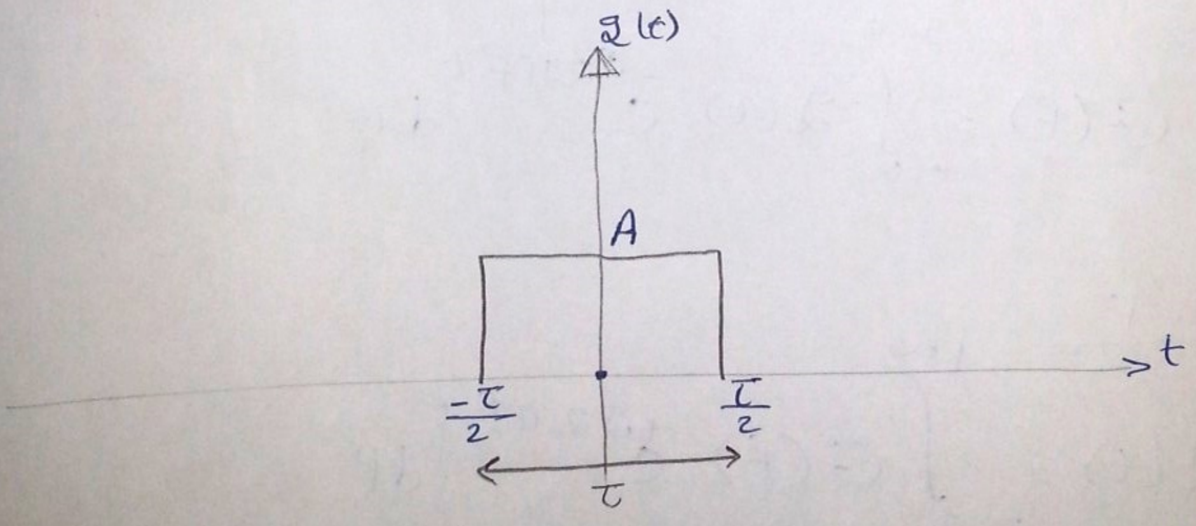
← في أي معادلة هنحتاج نحدد :-

١- إرتفاع المستطيل . ٢- مركز المستطيل ٣- عرض المستطيل

٣-

← في المثال ده :-

A ← إرتفاع المستطيل . τ ← عرض المستطيل



← لتحديد مركز الـ rect :-

① نتأكد أن البسط هو $1*t$ وفي حالة أن t مضروبة في أي رقم نقسم عليه (بسط ومقام)

② نساوي البسط بالصفر للحصول على المركز.

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-J2\pi f t} \cdot dt.$$

$$= \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot e^{-J2\pi f t} \, dt$$

$$= A \cdot \frac{e^{-J2\pi f t}}{(-J2\pi f)} \Bigg|_{-\frac{J}{2}}^{\frac{J}{2}}$$

$$= \frac{A}{(-J2\pi f)} \left[e^{-J2\pi f \frac{\tau}{2}} - e^{+J2\pi f \frac{\tau}{2}} \right]$$

$$= \frac{A}{-J2\pi f} \left[e^{-J\pi f \tau} - e^{+J\pi f \tau} \right]$$

<u>Note</u>

$$\cos\theta = \frac{e^{J\theta} + e^{-J\theta}}{2}, \quad \sin\theta = \frac{e^{J\theta} - e^{-J\theta}}{2J}$$

$$= \frac{A}{\pi f}\left[\frac{e^{+j\pi f\tau} - e^{-j\pi f\tau}}{2j}\right]$$

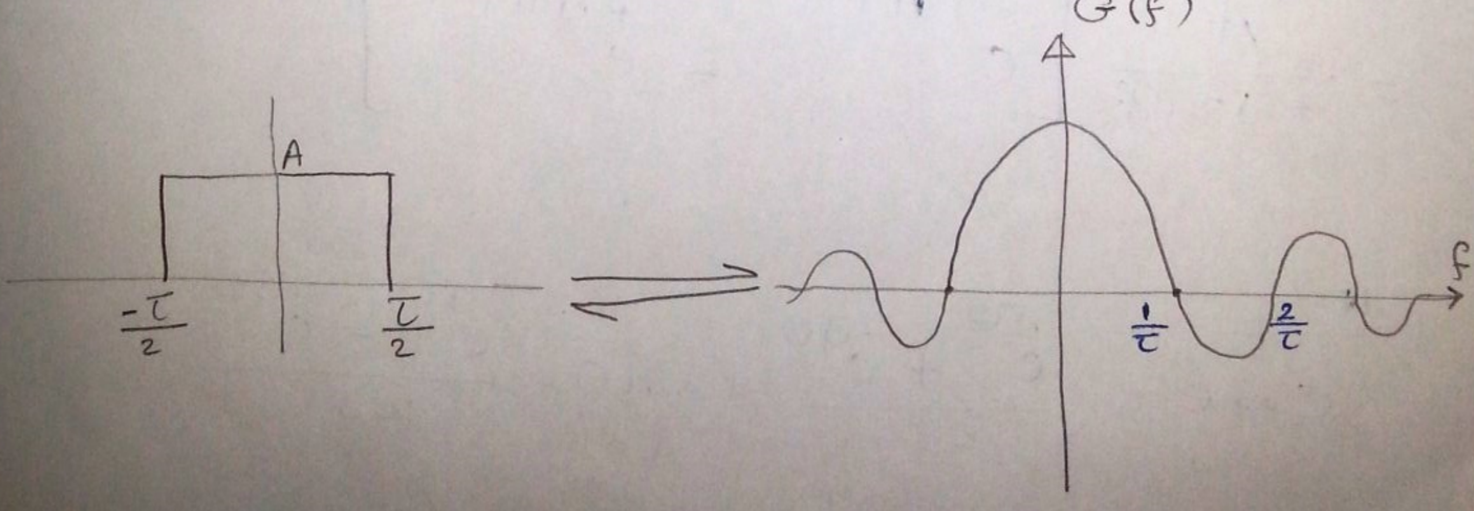
$$G(f) = \frac{A}{\pi f}\cdot \sin(\pi f \tau) \qquad \text{sinc}\, x = \frac{\sin(\pi x)}{\pi x}$$

$$G(f) = \frac{A\tau}{\pi f \tau} \sin(\pi f \tau)$$

$$G(f) = A\tau\ \text{sinc}(f\tau)$$

<u>Note</u>

$$A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\ \text{sinc}(f\tau)$$



$$\operatorname{sinc} x = \frac{\sin(\pi x)}{\pi x}$$

$$\operatorname{sinc}(0) = \frac{\sin(0)}{0} = 1 \quad , \lim_{x \to 0} \frac{\sin x}{x} = 1$$

x integer $\longrightarrow \operatorname{sinc}(x) = 0$

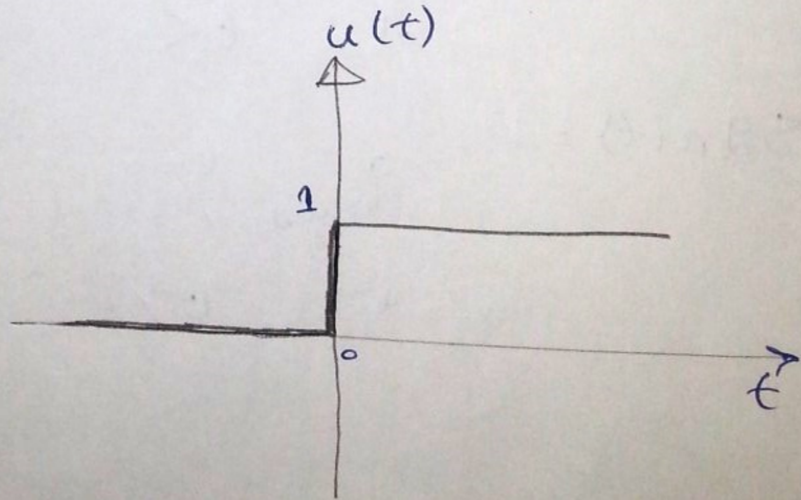
$x = 0$ ماعدا

$f\tau = 1, 2, 3 \ldots \longrightarrow f = \frac{1}{\tau}, \frac{2}{\tau}, \ldots$

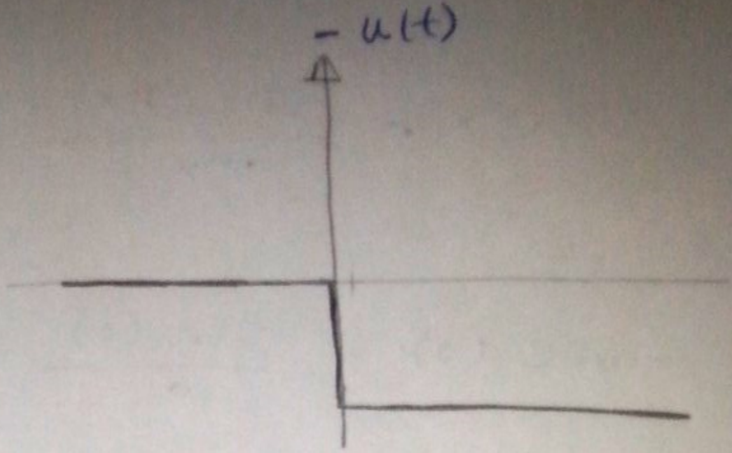
Some important functions:-

① unit step functions:-

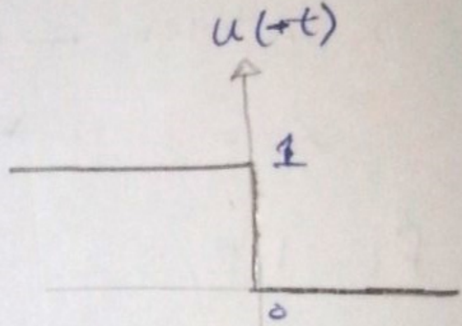
$$u(t) = \begin{cases} 0 & , t < 0 \\ \frac{1}{2} & , t = 0 \\ 1 & , t > 0 \end{cases}$$



$$-u(t) \begin{cases} 0 &, t<0 \\ \frac{-1}{2} &, t=0 \\ -1 &, t>0 \end{cases}$$

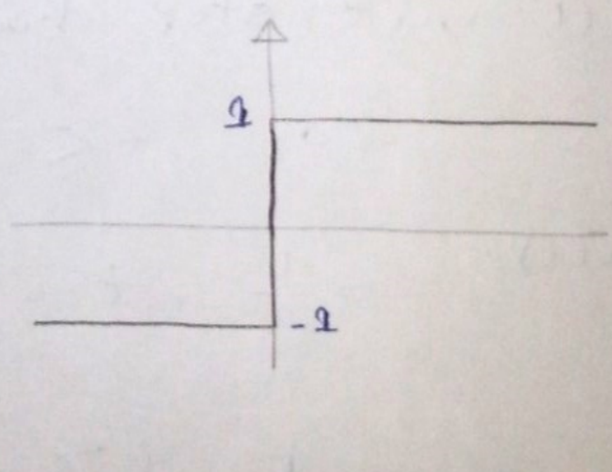


u(-t)



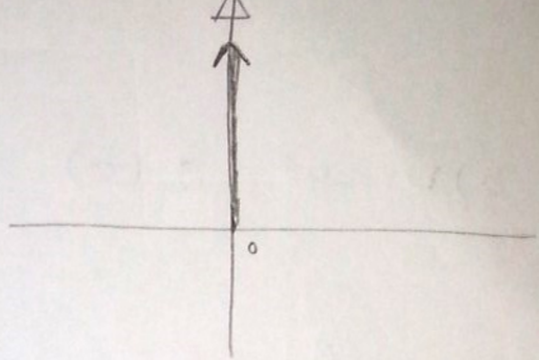
$\boxed{2}$ signum function sgn(t)

$$\mathrm{sgn}(t) = \begin{cases} -1 & t<0 \\ 0 \ \text{avg} & t=0 \\ +1 & t>0 \end{cases}$$



$\boxed{3}$ Delta Fn. $\delta(t)$

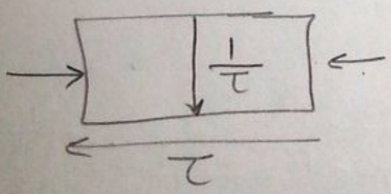
$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{o.w} \end{cases}$$



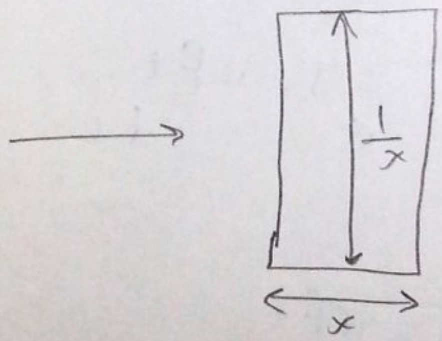
→ Area under $\delta(t)$ equals 1

$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

Note



Area = 1



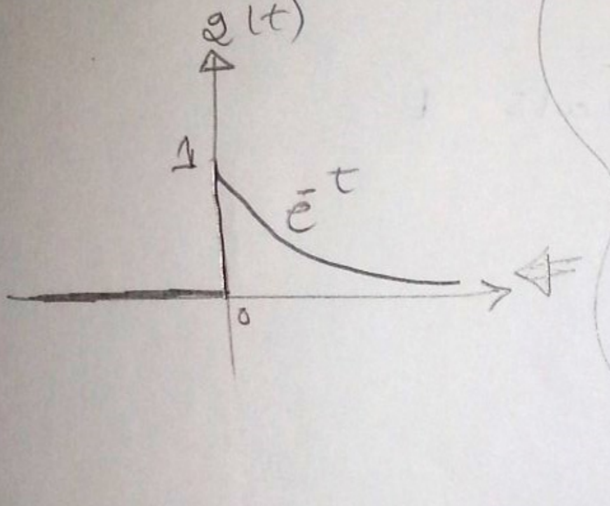
Area = 1

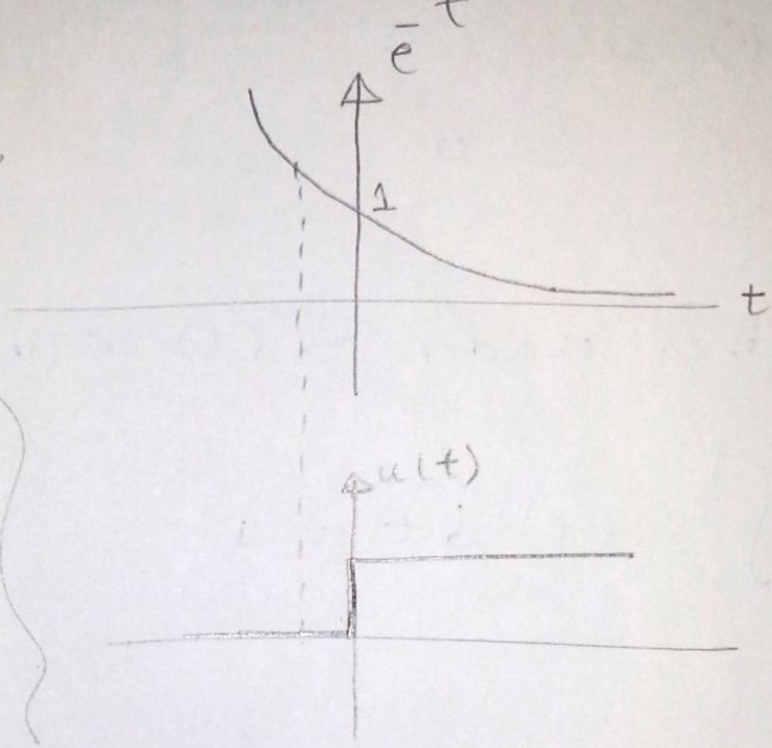
$$\therefore \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$\boxed{2}$ find F.T for $g(t) = e^{-t} \cdot u(t)$

sol

a) Draw $g(t)$





b)

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \, dt$$

sgn
rect
u(t)

لا يكتبوا في التكامل

$$= \int_{0}^{\infty} e^{-t} \cdot e^{-j2\pi f t} \, dt$$

تأثير u(t)

$$G(f) = \int_0^{\infty} e^{-t(1+J2\pi f)}\, dt$$

$$= \left. \frac{e^{-t(1+J2\pi f)}}{-(1+J2\pi f)} \right|_0^{\infty}$$

$$= \frac{1}{-(1+J2\pi f)} \left[\underbrace{e^{-\infty}}_{0} - \underbrace{e^{0}}_{1} \right]$$

$$G(f) = \frac{1}{1+J2\pi f}$$

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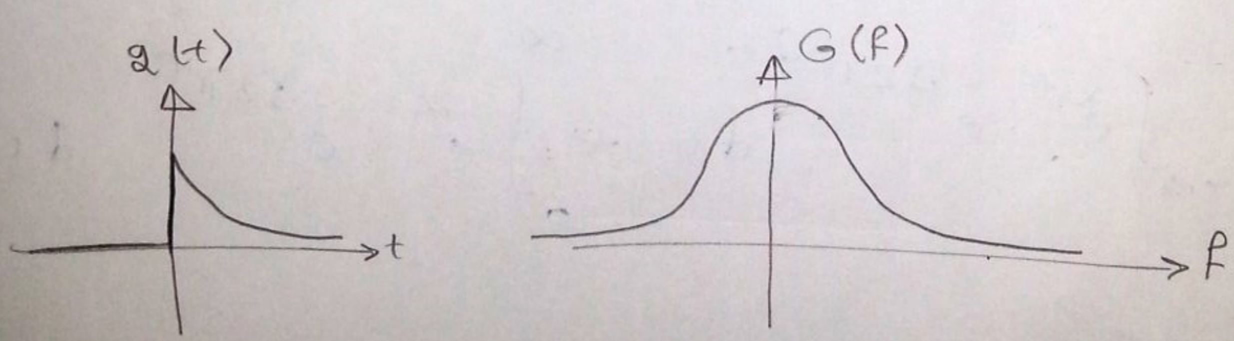
* $e^{-t} \cdot u(t) \rightleftharpoons \dfrac{1}{1+J2\pi f}$



$$|G(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$

$G(f) \longrightarrow$ Sometimes Complex

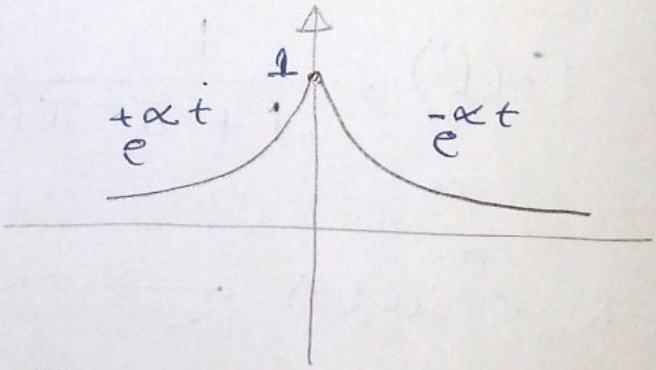
Find F.T of $g(t) = e^{-\alpha (t)}$

<u>Sol</u>

$$|t| = \begin{cases} +t & t \geq 0 \\ -t & t > 0 \end{cases}$$

$$g(t) = \begin{cases} e^{-\alpha t} & t > 0 \\ 1 & t = 0 \\ e^{+\alpha t} & t > 0 \end{cases}$$

$$G(f) = \int_{-\infty}^{+\infty} g(t) \cdot e^{-J 2\pi f t} \, dt$$



$$= \int_{-\infty}^{0} e^{\alpha t} \cdot e^{-J 2\pi f t} \, dt + \int_{0}^{\infty} e^{-\alpha t} \cdot e^{-J 2\pi f t} \, dt$$

$\cancel{\int_{-\infty}^{0} e^{\alpha t}}$

$$= \int_{-\infty}^{0} e^{t(\alpha - J2\pi f)}\,dt + \int_{0}^{\infty} e^{-t(\alpha + J2\pi f)}\,dt$$

$$= \left.\frac{e^{t(\alpha - J2\pi f)}}{\alpha - J2\pi f}\right|_{-\infty}^{0} + \left.\frac{e^{-t(\alpha + J2\pi f)}}{-(\alpha + J2\pi f)}\right|_{0}^{\infty}$$

$$= \frac{1}{\alpha - J2\pi f}\left(\underbrace{e^{0}}_{1} - \underbrace{e^{\infty}}_{0}\right) + \frac{1}{-(\alpha + J2\pi f)}\left(\underbrace{e^{-\infty}}_{0} - \underbrace{e^{0}}_{1}\right)$$

$$\boxed{G(f) = \frac{1}{\alpha - J2\pi f} + \frac{1}{\alpha + J2\pi f}}$$

* Fourier transform Properties

هى مجموعة من الخواص تساعدنا لإيجاد (F.T) لدوال مجهولة بمعلومية دوال أخرى معلومة وليس عن طريق التكامل.

$\boxed{1}$ Superposition (Linearity)

Let $g_1(t) \rightleftharpoons G_1(f)$

$g_2(t) \rightleftharpoons G_2(f)$

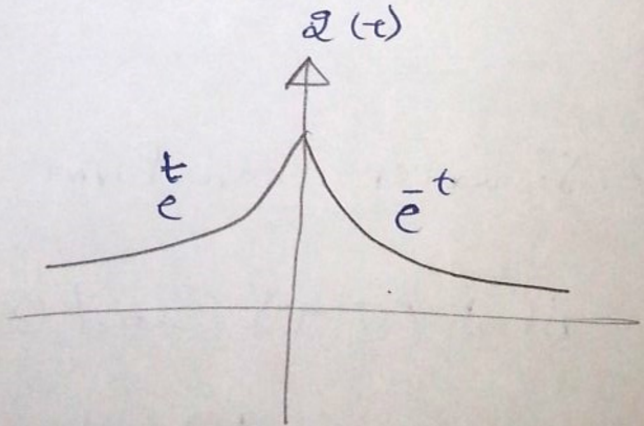
$$\therefore\ a \cdot g_1(t) \pm b\, g_2(t) \rightleftharpoons a \cdot G_1(f) \pm b \cdot G_2(f)$$

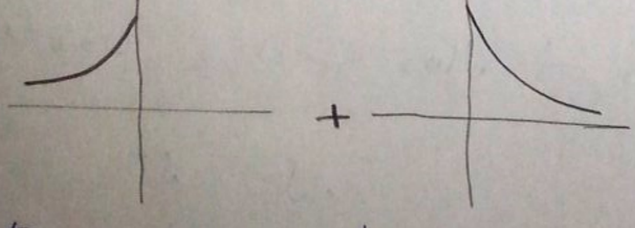
$$a \int g_1(t)\, e^{-j2\pi f t} \cdot dt \Rightarrow G_1(f) \cdot a$$

$$\pm b \int g_2(t)\, e^{-j2\pi f t} \cdot dt \Rightarrow G_2(f) \cdot b$$

$\boxed{4}$ Find F.T of $g(t) = e^{-|t|}$

$$g(t) = e^{-t} + e^{t}$$





$e^{t} \cdot u(-t)$ $\quad$ $e^{-t} \cdot u(t)$

$$g(t) = u(t) \cdot e^{-t} + e^{t} \cdot u(-t)$$

$$e^{-t} \cdot u(t) \rightleftharpoons \frac{1}{1 + J2\pi f}$$

$$e^{t} \cdot u(-t) \rightleftharpoons \frac{1}{1 - J2\pi f}$$

→ using Linearity

$$\therefore G(f) = \frac{1}{1 + J2\pi f} + \frac{1}{1 - J2\pi f}$$

$\boxed{2}$ Time scaling

if $\quad g(t) \rightleftharpoons G(f)$

then $g(a \cdot t) \rightleftharpoons \frac{1}{|a|} \cdot G\left(\frac{f}{a}\right)$

5] Find F.T for $x(t) = e^{-at} \cdot u(t)$

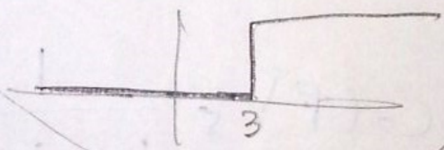
$$\because \underbrace{e^{-t} \cdot u(t)}_{x(t)} \rightleftharpoons \frac{1}{1+J2\pi f}$$

$u(t)$ ما بداخل القوس نساوية بالصفر

$t=0$

للحصول على النقطة التي حدث عندها (step)

ex $u(t-3)$



using time scaling

$$e^{-at} \cdot u(t) \rightleftharpoons \frac{1}{|a|} \frac{1}{1+J2\pi \frac{f}{a}}$$